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Transient mass transfer from an isothermal vertical flat plate

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Abstract—Transient mass transfer from a vertical, isothermal flat plate arising from buoyancy forces created by a concentration gradient is investigated assuming that all the fluid properties are constant except for the concentration-dependent body force. The solution is dependent on two parameters, namely, the Prandtl number, *Pr*, and the Schmidt number, *Sc*. An analytical solution is presented which is valid at small values of time and this solution is extended to larger values of time by using a modified Crank–Nicolson scheme. At large values of time this scheme becomes unstable and a specialized numerical scheme is then employed in order to extend the computations until the steady-state situation has been reached. Results have been obtained for a wide range of values of the Prandtl and Schmidt numbers but since all these show similar behaviours only the results with $Pr = Sc = 1$ are presented.

1. INTRODUCTION

THE FREE convection from a heated vertical plate was first examined by Lorenz [1] but Schmidt and Beckmann [2] showed that his assumptions were invalid. In the paper by Schmidt and Beckmann both theoretical and experimental results were presented and this work forms the basis of many subsequent studies of steady free convection boundary-layer flows past semi-infinite vertical plates.

Since these early papers there have been numerous attempts at unsteady free convection problems and in particular the determination of the velocity and temperature of a viscous fluid in the laminar boundary layer near a semi-infinite vertical flat plate whose temperature, at time $t' = 0$, is suddenly increased and maintained at a constant value still remains an open question. If x' is the distance measured along the plate from the leading edge the $\tau = t/x^{1/2}$ (where x and t are the non-dimensional distance and time, respectively), the solution for small values of τ is that as described by Illingworth [3] whilst the ultimate steady state is that as given by Ostrach [4]. Siegel [5], using a Pohlhausen technique, obtained a 'theoretical' solution and explained the physical phenomena. That is at a finite distance from the leading edge the flow develops as if the plate were infinite in extent but due to the wave-like nature of the unsteady boundary-layer equations a finite time elapses before the leading edge influences the flow development at that station and the transition to the classical steady-state solution takes place. These essential features of the flow have been confirmed experimentally by Goldstein and Eckert [6] and Gebhart *et al.* [7]. Gebhart [8] discusses the leading edge effect and uses an approximate method of solution to compare his experimental results. Explaining analytically and numerically the transition from the Illingworth to the Ostrach solution has proved very difficult.

Goldstein and Briggs [9] suggested that the leading edge signal penetrates a distance

$$x'_p = \max\left[\int_0^{t'} u'(y', t'_1)\, dt'_1\right] \tag{1}$$

whereas Brown and Riley [10] suggest the slightly greater value of

$$x'_p = \int_0^{t'} \max\,[u'(y', t'_1)]\, dt'_1. \tag{2}$$

In these expressions y' is the coordinate normal to the vertical plate, u' the x' component of the velocity of the fluid and the maximization is with respect to y'.

Hellums and Churchill [11] and Carnham *et al.* [12] have solved this problem numerically but their results indicate that the effect of the leading edge propagates at a speed which is in excess of the maximum one-dimensional (1-D) unsteady speed. Further their results were shown by Ingham [13] to be very dependent on the mesh size and the solution as the mesh size tends to zero was not clear. Other numerical solutions which are either unsatisfactory or show a departure from the unsteady 1-D solution earlier than that given by either equation (1) or (2) were reported by Brown and Riley [10]. Ingham [14] used four quite different numerical methods to solve this problem. He found that all the results showed:

(a) a departure from the unsteady 1-D solution before the theoretically predicted time,

(b) as the mesh size decreases the transition from the unsteady solution to the steady-state solution is quicker but not by means of a smooth transition.

More recently Nicholas [15] has investigated this problem using an alternative method of solution but his conclusions are similar to those found by Ingham [14].

Using techniques similar to those mentioned above several related problems have been successfully solved numerically with some analytical support. For example the determination of the motion of a viscous fluid past a semi-infinite flat plate which is impulsively set into

NOMENCLATURE

c'	concentration
c	dimensionless concentration, $(d' - c_\infty)/(c_w - c_\infty)$
D	diffusion coefficient
f	dimensionless stream function, $x^{-3/4}\psi$
g	acceleration due to gravity
Gr	Grashof number, $\beta g L^3(T_w - T_\infty)/v^2$
$h(\eta, \tau)$	dimensionless concentration
$H(\eta)$	dimensionless concentration as $\tau \to \infty$
L	typical length of flat plate
P	$-\frac{3}{4}f$
Pr	Prandtl number, v/α
q	$1 - \tau f/2$
Sc	Schmidt number, v/D
Sh_x	local Sherwood number
t'	time
t	nondimensional time, Ut'/L
u'	x-velocity component
u	dimensionless x-velocity component, u'/U
U	a quantity with dimensions of speed, $[\beta g L(T_w - T_\infty)]^{1/2}$
v	y-velocity component
v'	dimensionless y-velocity component, v'/U
x'	spatial coordinate along the plate
x	dimensionless spatial coordinate along the plate, x'/L
y'	spatial coordinate normal to the plate
y	dimensionless spatial coordinate normal to the plate, $y'\,Gr^{1/4}/L$.

Greek symbols

α	thermal diffusivity
β	volumetric coefficient of thermal expansion
β^*	volumetric coefficient of expansion with concentration
ζ	independent variable, $\eta(Sc/\tau)^{1/2}/2$
η	independent variable, $y/x^{1/4}$
$\Delta\eta$	dimensionless spatial step
μ	viscosity
v	kinematic viscosity, μ/ρ
ρ	density
τ	independent variable, $t/x^{1/2}$
τ_c	value of τ when $1 - \tau f_{max}/2 = 0$
$\Delta\tau$	dimensionless time step
ψ'	stream function
ψ	dimensionless stream function, $\psi'\,Gr^{1/4}/(UL)$.

Subscripts

m	station where asymptotic solution may be assumed to hold
w	at the surface of the plate
∞	free stream conditions.

motion with a constant velocity parallel to itself has been investigated by many authors, notably Stewartson [16, 17], Hall [18], Dennis [19] and Watson (in an appendix to Dennis). In addition the author found that all the four methods attempted on the free convection problem without success gave very good agreement with those already published on this problem. Other similar problems where no difficulties have arisen are:

(a) the unsteady heat transfer for the boundary layer flow over a flat plate [20],
(b) the unsteady heat transfer in impulsive Falkner–Skan flows [21],
(c) the boundary layer in a shock tube [22],
(d) the flow past an impulsively started semi-infinite flat plate in the presence of a magnetic field [23].

In all these problems the leading edge disturbance is first detected at the outer edge of the boundary layer whereas in the free convection problem it should occur within the boundary layer.

Callahan and Marner [24] extended the work of Hellums and Churchill [11] and Carnham *et al.* [12] to deal with the effects of transient mass transfer in addition to the unsteady free convection effects on an isothermal vertical flat plate. However, these results show the same unsatisfactory nature as the results with no mass transfer.

In this paper the problem is considered in which at $t' < 0$ the steady free convective fluid flow and temperature, as described by Ostrach [4], from a heated vertical plate is fully established. The concentration of the fluid being c_∞ everywhere and the plate at a constant temperature T_w whilst the surrounding fluid is maintained at a constant temperature T_∞. At time $t' = 0$ the concentration of the fluid at the plate is suddenly changed to c_w, thereby setting up a time-dependent concentration boundary layer. In this paper it is assumed that:

(a) the volumetric coefficient of expansion with concentration is small so that there is no variation in the velocity and thermal quantities,
(b) the fluid properties are constant except for the body force terms in the concentration equations,
(c) no chemical reactions take place,
(d) viscous dissipation is negligible.

Under these assumptions the growth of the concentration boundary layer may be studied by solving a relatively simple partial differential equation for the concentration. The object of the present paper is to study the transition of the boundary layer growth from the initial state to the final state.

If assumption (a) is not made then the problem would remain as complex as that studied by Callahan and Marner [24] for which there is no satisfactory explanation. Making this assumption therefore reduces the problem to determining the concentration only. However, the disturbance from the leading edge is now transmitted fastest through the boundary layer from within rather than at the outer edge of the boundary layer. It is exactly this mechanism which occurs in the yet unresolved problem of the unsteady free convective boundary layer flow and it is hoped that the present work will throw some light onto this very complex problem.

In this paper results will be presented for fluids in which both the Prandtl and Schmidt numbers are unity although results have been obtained, with no difficulties, for a wide range of values of these parameters. An analytical solution which is valid for small values of τ, as a series in $\tau^{1/2}$, where

$$\tau = [g\beta(T_w - T_\infty)]^{1/2} t'/x'^{1/2},$$

is presented along with numerical results for the complete range of values of τ.

2. EQUATIONS

The coordinates (x', y') are measured along the semi-infinite flat plate and normal to it, respectively, with the origin at the leading edge. The x'-axis being parallel to the direction of gravity but directed upwards. It is assumed that for $t' < 0$ the steady-state free convective boundary layer is already established. Subsequent changes in the density, ρ, and viscosity, $\rho\nu$, are assumed to be small and the volumetric coefficient of expansion with concentration, β^*, negligible so that the velocity and thermal boundary layers remain independent of time. Based on these assumptions the continuity, momentum, energy and concentration equations, become [24]

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \tag{3}$$

$$u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} = \nu\frac{\partial^2 u'}{\partial y'^2} + \beta g(T' - T_\infty) \tag{4}$$

$$u'\frac{\partial T'}{\partial x'} + v'\frac{\partial T'}{\partial y'} = \alpha\frac{\partial^2 T'}{\partial y'^2} \tag{5}$$

$$\frac{\partial c'}{\partial t'} + u'\frac{\partial c'}{\partial x'} + v'\frac{\partial c'}{\partial y'} = D\frac{\partial^2 c'}{\partial y'^2} \tag{6}$$

where c' is the concentration, T' the temperature, α the thermal diffusivity of the fluid, D the diffusion coefficient, β the volumetric coefficient of thermal expansion and g the acceleration due to gravity. Following Ostrach [4] dimensionless variables are introduced

$$\left.\begin{aligned} x &= \frac{x'}{L}, \quad y = \frac{Gr^{1/4}\, y'}{L}, \quad t = \frac{Ut'}{L}. \\ \psi &= \frac{Gr^{1/4}\,\psi'}{UL}, \quad T = \frac{T' - T_\infty}{T_w - T_\infty}, \quad c = \frac{c' - c_\infty}{c_w - c_\infty} \end{aligned}\right\} \tag{7}$$

where ψ' is the stream function such that the continuity equation (3) is satisfied, i.e.

$$u' = \frac{\partial\psi'}{\partial y'}, \quad v' = -\frac{\partial\psi'}{\partial x'}. \tag{8}$$

Here L is a typical length along the plate, $U = [g\beta L(T_w - T_\infty)]^{1/2}$ is quantity with the dimensions of speed and $Gr = \beta g L^3(T_w - T_\infty)/\nu^2$ is the Grashof number (it is assumed that $Gr \gg 1$ in order to use the boundary layer approximations in equations (3)–(6)). The Prandtl number Pr is ν/α and the Schmidt number Sc is ν/D.

At $t' = 0$ the concentration of the fluid at the plate is impulsively changed from c_∞ to c_w. Thus equations (3)–(6) must be solved subject to the boundary conditions

$$\left.\begin{aligned} &u = v = 0, \quad T = 1, \quad y = 0, x \geqslant 0, \text{ all } t \\ &u \to 0, \quad T \to 0, \quad y \to \infty, x \geqslant 0, \text{ all } t < 0 \end{aligned}\right\} \tag{9}$$

$$\left.\begin{aligned} &c = 0, \quad \text{all } y, \text{ all } x, t < 0 \\ &c = 1, \quad y = 0, x \geqslant 0, t \geqslant 0 \\ &c = 0, \quad y \to \infty, x \geqslant 0, t \geqslant 0. \end{aligned}\right\} \tag{10}$$

In order to reduce the number of independent variables from three to two the following transformations are made

$$\left.\begin{aligned} &\psi = x^{3/4} f(\eta), \quad T = g(\eta), \quad c = h(\eta, \tau) \\ &\eta = y/x^{1/4}, \quad \tau = t/x^{1/2}. \end{aligned}\right\} \tag{11}$$

Substituting these into equations (4)–(6) gives rise to the differential equations

$$f''' + \tfrac{3}{4}ff'' - \tfrac{1}{2}f'^2 + g = 0 \tag{12}$$

$$g'' + \tfrac{3}{4}Pr\, fg' = 0 \tag{13}$$

$$\left(1 - \frac{\tau f'}{2}\right)\frac{\partial h}{\partial \tau} - \tfrac{3}{4}Sc\, f\frac{\partial h}{\partial \eta} = \frac{\partial^2 h}{\partial \eta^2} \tag{14}$$

where primes denote differentiation with respect to η.

Boundary conditions (9) and (10) now reduce to

$$\left.\begin{aligned} &f(0) = f'(0) = 0, \quad g(0) = 1 \\ &f'(\infty) = 0, \quad g(\infty) = 0 \end{aligned}\right\} \tag{15}$$

$$\left.\begin{aligned} &h = 0, \quad \eta \geqslant 0, \quad \tau < 0 \\ &h = 1, \quad \eta = 0, \quad \tau \geqslant 0 \\ &h = 0, \quad \eta \to \infty, \quad \tau \geqslant 0. \end{aligned}\right\} \tag{16}$$

Equations (12) and (13) subject to boundary conditions (15) are the classical steady-state boundary layer equations for the free-convection flow over a semi-infinite flat plate as described by Ostrach [4] and the solution is well known. Further, equation (14) with boundary conditions (16) is very similar to that considered by Riley [25] when considering the unsteady heat transfer for flow over a flat plate.

The variables η and τ and equation (14) are the most appropriate for studying the final decay to the steady-state solution. However, in the initial stages of the growth of the diffusion boundary layer, when the effects of species diffusion is important, a new independent variable is taken

$$\xi = y'/2(Dt')^{1/2} = \eta(Sc/\tau)^{1/2}/2. \quad (17)$$

Written in terms of the variables (ξ, τ) the concentration equation (14) becomes

$$\frac{\partial^2 h}{\partial \xi^2} + \frac{\partial h}{\partial \xi}\left[2\xi\left(1 - \frac{\sqrt{\tau}\, Sc}{4}\frac{\partial f}{\partial \xi}\right) + \frac{3}{2}\sqrt{\tau}\, Sc\, f\right] - \tau\left[4 - \sqrt{\tau}\, Sc\frac{\partial f}{\partial \xi}\right]\frac{\partial h}{\partial \tau} = 0 \quad (18)$$

and boundary conditions (16) become

$$\left.\begin{aligned} h &= 0, \quad \xi \geqslant 0, \quad \tau < 0 \\ h &= 1, \quad \xi = 0, \quad \tau \geqslant 0 \\ h &\to 0, \quad \xi \to \infty, \quad \tau \geqslant 0. \end{aligned}\right\} \quad (19)$$

In all that follows both the Prandtl and Schmidt numbers are set to unity for convenience. The analysis and numerical work has been performed for a range of values of these parameters but only the results for $Pr = Sc = 1$ are presented in this paper.

3. SOLUTION OF THE DIFFUSION EQUATION FOR SMALL VALUES OF τ

Since the diffusion boundary layer grows within the free convection boundary layer then for sufficiently small values of τ the 'stream function' f may be replaced by its series expansion which is valid near the wall when developing the solution for small τ. It can easily be shown that

$$f = \frac{1}{2}a\eta^2 - \frac{1}{6}\eta^3 - \frac{b}{24}\eta^4 \ldots \quad (20)$$

where

$$a = f''(0) \quad \text{and} \quad b = g'(0). \quad (21)$$

Numerical integration of equations (12) and (13) subject to boundary conditions (15) gives

$$a = 0.90819, \quad b = -0.40103. \quad (22)$$

Substituting transformation (17) into expression (20) which in turn is substituted into equation (18) gives

$$\frac{\partial^2 h}{\partial \xi^2} + \frac{\partial h}{\partial \xi}\left[2\xi + a\xi^2\tau^{3/2} + \frac{1}{3}b\xi^4\tau^{5/2}\right] = 4\tau\frac{\partial h}{\partial \tau} + O(\tau^3). \quad (23)$$

This suggests that

$$h(\xi, \tau) = h_0(\xi) + \tau^{3/2}h_1(\xi) + \tau^{5/2}h_2(\xi) + O(\tau^3) \quad (24)$$

which on substitution into equation (23) and equating powers of τ gives

$$\left.\begin{aligned} h_0'' + 2\xi h_0' &= 0 \\ h_1'' + 2\xi h_1' - 6h_1 &= -a\xi^2 h_0' \\ h_2'' + 2\xi h_2' - 10h_2 &= -\tfrac{1}{3}b\xi^4 h_0' \end{aligned}\right\} \text{etc.} \quad (25)$$

and the boundary conditions (19) become

$$\left.\begin{aligned} h_0(0) = 1, \quad h_1(0) = h_2(0) = 0 \\ h_0(\infty) = h_1(\infty) = h_2(\infty) = 0. \end{aligned}\right\} \quad (26)$$

The solution of equations (25) subject to boundary conditions (26) gives

$$\left.\begin{aligned} h_0(\xi) &= \operatorname{erfc}\xi \\ h_1(\xi) &= \frac{a}{24\sqrt{\pi}}\left\{3\xi^2 e^{-\xi^2} + \frac{\sqrt{\pi}}{2}(3\xi + 2\xi^3)\operatorname{erfc}\xi\right\} \\ h_2(\xi) &= -\frac{b}{64\sqrt{\pi}}\{\xi^2 + 2\xi^4\} - \frac{b}{96}\left\{\frac{3}{4}\xi + \xi^3 - \frac{1}{5}\xi^5\right\}\operatorname{erfc}\xi. \end{aligned}\right\} \quad (27)$$

By retaining more terms in equation (23) further terms in expansion (24) may be obtained.

4. NUMERICAL SOLUTION OF THE DIFFUSION EQUATION FOR $\tau \leqslant 2/f'_{max}$

The analytical solution presented in Section 3 provides the initial condition which is required to start a step-by-step solution of the parabolic partial differential equation (18), provided that

$$q = 4 - \sqrt{\tau}(\partial f/\partial \xi) > 0.$$

Thus a modified Crank–Nicolson method is used until $q = 0$, i.e. when

$$\tau = 2/\{f'(\eta)\}_{max} = \tau_c,$$

say. For $Pr = 1$ the value of τ_c is approximately 3.99. It should be remembered that in this problem f' attains its maximum value within the interior of the boundary layer.

5. NUMERICAL SOLUTION OF THE DIFFUSION EQUATION FOR $\tau \leqslant 2/f'_{max}$

Since equation (14) is more appropriate than equation (18) at large values of τ so that at $\tau = \tau_c$ one reverts back to equation (14) and the method of solution is very similar to that described by Dennis [19]. Equation (14) can be written as

$$\frac{\partial^2 h}{\partial \eta^2} + p\frac{\partial h}{\partial \eta} = q\frac{\partial h}{\partial \tau} \quad (28)$$

where $p = -\frac{3}{4}f$. A finite-difference grid is constructed with sides parallel to the directions of η and τ. If $\Delta\eta$ and $\Delta\tau$ are the grid sizes in these directions, respectively, an implicit scheme is obtained by approximating the LHS of equation (28) by central differences and the RHS by backward differencing if $q > 0$ and forward differencing if $q < 0$. Thus the finite-difference representation of equation (28) used was

$$[1+\tfrac{1}{2}\Delta\eta p(\eta,\tau)]h(\eta+\Delta\eta,\tau)$$
$$+[1-\tfrac{1}{2}\Delta\eta p(\eta,\tau)]h(\eta-\Delta\eta,\tau)+\frac{(\Delta\eta)^2}{\Delta\tau}q(\eta,\tau)X$$
$$=\left[2+\frac{(\Delta\eta)^2}{\Delta\tau}|q(\eta,\tau)|\right]h(\eta,\tau) \quad (29)$$

where

$$X=\begin{Bmatrix} h(\eta,\tau-\Delta\tau), & q>0 \\ -h(\eta,\tau+\Delta\tau), & q<0. \end{Bmatrix} \quad (30)$$

It is not now possible to solve equation (29) by a step-by-step procedure in τ because of the forward influence implied by equation (30) when $q<0$. The matrix associated with the finite-difference equations (29) extend to all grid points in the region $0 \leqslant \eta \leqslant \eta_m$, $\tau_c \leqslant \tau \leqslant \tau_m$ where η_m and τ_m are sufficiently large values of η and τ for the asymptotic nature of the solution to have been reached. It is necessary to specify the boundary conditions on h as $\tau \to \infty$. This is the steady-state solution which is obtained by solving equation (14) with $(\partial h/\partial\tau) \equiv 0$, this gives the solution $H(\eta)$ which satisfies

$$\frac{d^2H}{d\eta^2}+\frac{3}{4}f\frac{dH}{d\eta}=0, \quad H(0)=1, \quad H(\infty)=0. \quad (31)$$

The complete boundary conditions are therefore

$$\left.\begin{array}{l} h=1 \quad \text{on} \quad \eta=0, \quad h\to 0 \quad \text{as} \quad \eta\to\infty, \quad \tau>\tau_c \\ h\to H(\eta) \quad \text{as} \quad \tau\to\infty \\ h=h(\eta,\tau_c) \quad \text{when} \quad \tau=\tau_c. \end{array}\right\} \quad (32)$$

The finite-difference equations (29) subject to boundary conditions (32) can now be solved by successive over-relaxation. Since the matrix associated with equation (29) is always diagonally dominant, if $\Delta\eta$ is taken sufficiently small, this guarantees the convergence of the iterative scheme.

6. RESULTS

Figure 1 shows the variation of the non-dimensional component of velocity parallel to the plate $f'(\eta)$ $(=(\partial\psi'/\partial y')/U)$ as a function of η. This shows that the maximum velocity of Uf'_{max} now occurs within the boundary layer and $\tau_c = 2/f'_{max} \simeq 3.99$.

Equation (18) subject to boundary conditions (19) was solved using the Crank–Nicolson method up to $\tau = \tau_c$ with various values of locations of the infinity condition on ξ, ξ_m say. It was found that $\xi_m = 4.0$ was sufficiently large. Also various values of $\Delta\eta$ and $\Delta\tau$ were used and $\Delta\eta = 0.1$ and $\Delta\tau = 0.1$ were found to give accurate results.

In order to check the numerical solution with the analytical solution one computes the local Sherwood

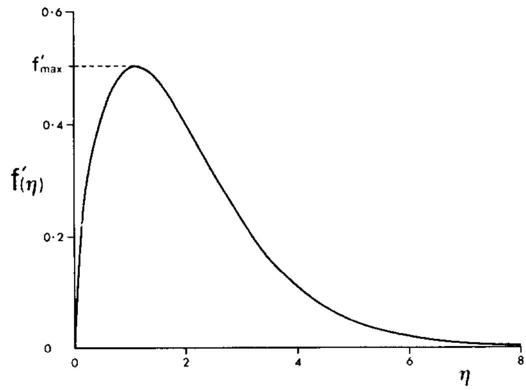

FIG. 1. The non-dimensional component of velocity parallel to the plate as a function of η.

number, Sh_x, which is given by

$$Sh_x = -\left(\frac{\partial c}{\partial y}\right)_{y=0} x\ Gr^{1/4} \quad (33)$$

and hence

$$\left(\frac{\partial h}{\partial\eta}\right)_{\eta=0} = -Sh_x/\{x^{3/4}\ Gr^{1/2}\}. \quad (34)$$

From equation (27) one obtains

$$-\left(\frac{\partial h}{\partial\eta}\right)_{\eta=0} = \frac{1}{\sqrt{\tau}}[0.5642+0.02838\tau^{3/2} +0.00157\tau^{5/2}\ldots] \quad (35)$$

and this is tabulated in Table 1. Also shown in Table 1 are the numerically calculated values. It is seen that the analytical solution, which is valid for small values of τ, is in fact accurate for values of τ up to about 4.

Figure 2 shows the non-dimensional concentration profiles at various values of τ as a function of η. This shows, as one would expect, that the approach to the steady-state profile is much faster near the wall than at the outer edge of the boundary layer. Both Fig. 2 and

Table 1. The variation of $-\left(\frac{\partial h}{\partial\eta}\right)_{\eta=0}$ as a function of τ

τ	Two-term expansion equation (35)	Three-term expansion equation (35)	Numerical solution
0.5	0.8121	0.8125	0.8126
1.0	0.5926	0.5941	0.5943
1.5	0.5032	0.5041	0.5065
2.0	0.4557	0.4612	0.4612
2.5	0.4278	0.4376	0.4355
3.0	0.4109	0.4250	0.4205
3.5	0.4009	0.4201	0.4119
4.0	0.3956	0.4207	0.4070
4.5	0.3937	0.4256	0.4045
5.0	0.3942	0.4334	0.4033
5.5	0.3967	0.4441	0.4024
6.0	0.4006	0.4570	0.4019
7.0	0.4119	0.4887	0.4014
8.0	0.4265	0.5268	0.4012
steady			0.4010

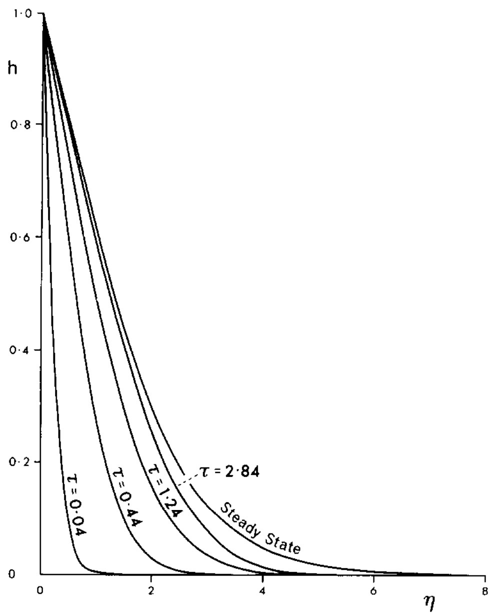

FIG. 2. The non-dimensional concentration profiles at various values of τ as a function of η.

Table 1 indicate that for $\tau \sim \tau_c$ the steady-state solution has almost been reached. An asymptotic solution at large values of $\tau(\tau \gg \tau_c)$ could be obtained in a manner similar to that described by Riley [25]. However, since the steady state has almost been achieved at $\tau \sim \tau_c$ the validity of this solution could not be checked.

The results presented in this paper clearly show that smooth transition from the unsteady to the steady-state situation. Thus the fact that the disturbance is travelling fastest within the boundary layer does not cause any difficulty in this problem. The situation in the suddenly heated vertical flat plate is that the governing equations are two coupled equations, akin to that given in equation (14), with the disturbances travelling fastest within the boundary layer. Ingham [23] showed that in a similar problem the coupling of the equation did not cause any difficulty in matching the small and large time solutions and in this paper the disturbance travelling within the boundary layer has caused no numerical difficulties. It must therefore be concluded that it is the coupling of these two effects that makes the free convection problem so intractable.

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TRANSFERT MASSIQUE VARIABLE PAR UNE PLAQUE PLANE ISOTHERME ET VERTICALE

Résumé—Le transfert massique variable à partir d'une plaque plane isotherme et verticale, par suite de forces d'Archimède créés par un gradient de concentration, est étudié en supposant que les propriétés de fluide sont constantes à l'exception de la dépendance des forces vis-à-vis de la concentration. La solution dépend de deux paramètres, le nombre de Prandtl *Pr* et le nombre de Schmidt *Sc*. Une solution analytique est présentée qui est valable pour les petites valeurs du temps et cette solution est étendue à de grandes valeurs du temps en utilisant un schéma modifié de Crank–Nicolson. Pour les grandes valeurs du temps ce schéma devient instable et un schéma numérique spécial est alors employé de façon à étendre les calculs jusqu'à la situation de régime permanent. Des résultats sont obtenus pour un large domaine de nombres de Prandtl et de Schmidt mais comme il y a similitude de comportement, seuls les résultats avec $Pr = Sc = 1$ sont présentés.

INSTATIONÄRER STOFFÜBERGANG AN EINER SENKRECHTEN, EBENEN, ISOTHERMEN PLATTE

Zusammenfassung—Der instationäre Stoffübergang an einer senkrechten, ebenen, isothermen Platte, der durch Auftriebskräfte infolge von Konzentrationsgradienten zustandekommt, wird untersucht. Dabei wird angenommen, daß alle Stoffeigenschaften des Fluids außer der konzentrationsabhängigen Auftriebskraft konstant sind. Die Lösung hängt von zwei Parametern ab, nämlich von der Prandtl-Zahl *Pr* und der Schmidt-Zahl *Sc*. Eine analytische Lösung wird vorgestellt, die für kurze Zeiträume gilt. Mit Hilfe des modifizierten Crank–Nicolson-Verfahrens wird die Gültigkeit dieser Lösung auf längere Zeiträume ausgedehnt. Für lange Zeiträume wird dieses Verfahren jedoch instabil und daher durch ein spezielles Rechenverfahren ersetzt, um die Berechnungen bis zum Erreichen des stationären Endzustands ausdehnen zu können. Die Ergebnisse umfassen einen großen Bereich der Prandtl- und der Schmidt-Zahl. Da aber alle Resultate ein ähnliches Verhalten zeigen, werden nur diejenigen mit $Pr = Sc = 1$ vorgestellt.

НЕСТАЦИОНАРНЫЙ ТЕПЛОПЕРЕНОС ОТ ИЗОТЕРМИЧЕСКОЙ ВЕРТИКАЛЬНОЙ ПЛОСКОЙ ПЛАСТИНЫ

Аннотация—Исследуется нестационарный теплоперенос от вертикальной изотермической плоской пластины за счет выталкивающих сил, создаваемых градиентом концентрации. Исследование проводится в предположении постоянства всех свойств жидкости за исключением зависящей от концентрации массовой силы. Решение определяется двумя параметрами: числами Прандтля и Шмидта. Представлено аналитическое решение, справедливое при малых значениях времени, которое с помощью модифицированной модели Крэнка–Никольсона обобщено на большие времена. При больших временах модель становится неустойчивой и тогда используется специальная численная схема для продолжения счета вплоть до стационарного состояния. Получены результаты для широкого диапазона изменения чисел Прандтля и Шмидта, но в силу их идентичности в статье приведены данные только для случая $Pr = Sc = 1$.